

# EE 261 The Fourier Transform and its Applications

## Fall 2025

### Course Information and Outline

**Instructor:**

Brad Osgood  
osgood@stanford.edu  
Office: Packard 271

**Course Assistants:**

Aqib Ahmed Syed aasyed@stanford.edu	Ridvan Yesiloglu ridvan@stanford.edu
--	---

**Administrative Assistant:**

Helen Lin  
helenlin@stanford.edu

---

**Course home page:** The official course website is on **Canvas**, and should be easy for you to track down. If you are registered on AXESS you are automatically registered on Canvas. Course materials (announcements, problem sets, etc) will be posted on the Canvas site.

**Lectures:** Class will be held T Th 10:30 - 11:50 AM in 420-040. Currently there are no plans to record the lectures.

There is also a discussion forum set up on **Ed**. Very useful for posting questions, answering questions, etc. This, too, can be accessed through the Canvas home page.

**Course Text** The book for the course is by your humble instructor: Brad G. Osgood, *Lectures on the Fourier Transform and its Applications*, published by the American Mathematical Society. The book is available through the Stanford Bookstore, the American Mathematical Society, and Amazon. A pdf of the book is posted on Canvas.

The book is also on reserve in the Terman Engineering library, along with several other references that you may find useful. Hardback books. How charming.

**Written work for the course:** There will be weekly problem sets, a midterm exam and a final exam. The problem sets will be assigned (via Canvas) on Thursdays and due the following Thursday. More on this, below.

- We will use Gradescope for submission and grading of the problem sets. You are responsible for scanning and submitting your work on the day it is due.

Grades for the course will be based on:

- Problem sets: 35%. *You can drop the lowest HW score.*
- Midterm: 25%.
  - The midterm will be scheduled for 90 minutes *outside of class time*. Details to be announced.
- Final: 40%.
  - The final exam is scheduled by the Registrar’s office. The date and time are TBA.

**Important: Software and use of AI** Most problem sets will include a problem using Matlab. We wrote problems with Matlab in mind because at the time it was the only real alternative. If you want to work in Python, Julia, Mathematica, or whatever, that’s perfectly fine. In fact, I’d appreciate someone making the translation, or allowing us to steal your code (with attribution!). Incidentally, I use Mathematica in my own work – I like the symbolic capabilities – so my Matlab (and Python) skills are weak.

**Obtaining Matlab et al** A wonderful thing has happened. You can now have access to Matlab and Simulink and Mathematica *at no charge*. Go to the Stanford Software Licensing Webstore.

**Using AI** My AI experience is limited, but I’ve been impressed – not to say bewildered – when I’ve tried it on various mathematical problems. Far from suppressing AI in the context of written assignments for the class, we expect and encourage its use. This includes turning in solutions that may, in part, derive from an AI session. That’s fine, but we expect you to make clear your use of AI.

Stanford provides access to several AI environments via the Stanford AI Playground:

<https://uit.stanford.edu/aisplayground>

For our part this is also an experiment to learn which kinds of problems lend themselves to AI and which don’t. (By the way, this should eliminate students asking for extensions; maybe all you have to do is type the question into ChatGPT and you don’t get an extension for that!)

**Students with Documented Disabilities:** Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty dated in the current quarter in which the request is made. Students should contact the OAE as soon as possible since timely notice is needed to coordinate accommodations. The OAE is located at 563 Salvatierra Walk; Phone: (650) 723-1066, URL <http://studentaffairs.stanford.edu/oea>

**Honor Code:** Be sure you are familiar with Stanford's Honor Code and Fundamental Standard. You can find the statements via the Registrar's web page, or directly at

<https://communitystandards.stanford.edu/student-conduct-process/honor-code-and-fundamental-standard>

Additional comments on the use (and abuse) of on-line material can be found here:

<https://communitystandards.stanford.edu/bja-guidance-remote-teaching-and-learning-environment>

---

**The course at a glance (taken from the Preface to the book)** The Fourier transform has it all. As a tool for applications it is used in virtually every area of engineering and science. In electrical engineering, methods based on the Fourier transform are used in all varieties of signal processing, from communications and circuit design to the analysis of imaging systems. In materials science, physics, chemistry and molecular biology it is used to understand diffraction and the crystal structures of compounds. There's even a field called Fourier Optics. In mathematics, Fourier series and the Fourier transform are cornerstones of the broad area known as harmonic analysis, and applications range from number theory to the modern formulation of the theory of partial differential equations. Lurking not so far beneath the surface are deep connections to groups and symmetry. Particularly widely used in practical applications is the discrete Fourier transform computed via the FFT (Fast Fourier Transform) algorithms. It is not an exaggeration to say that much of the increasingly digital world depends on the FFT.

Historically, Fourier analysis developed from employing sines and cosines to model periodic physical phenomena. This is the subject matter of Fourier series (our first topic), and here we learn that a complicated signal can be written as a sum of sinusoidal components, the simplest periodic signals.<sup>1</sup> Borrowing from musical terminology, where pure tones are single sinusoids and the frequency is the pitch, the component sinusoids are often called the harmonics. Complicated tones are sums of harmonics. In this way, with a periodic signal we associate a discrete set of frequencies — its spectrum — and the amount that each harmonic contributes to the total signal. If we know the spectrum and the amount that each harmonic contributes then we know the signal, and vice versa. We analyze the signal into its component parts, and we synthesize the signal from its component parts.

The Fourier transform arose as a way of analyzing and synthesizing nonperiodic signals. The spectrum becomes a continuum of frequencies rather than a discrete set. Through the Fourier transform, and its inverse, we now understand that:

*Every signal has a spectrum, and the spectrum determines the signal.*

This maxim surely ranks as one of the major secrets of the universe. A signal (often a time varying function, thus a representation in the “time domain”) and its Fourier transform (a function depending on the spectrum, thus a representation in the “frequency domain”) are equivalent in that one determines the other and we can pass back and forth between the two. The signal appears in different guises in the time domain and in the frequency domain, and this enhances the usefulness of both representations.

---

<sup>1</sup>Electrical engineers speak in terms of signals and mathematicians in terms of functions. As far as I'm concerned, the two terms are interchangeable. I'll use both.

“Two representations for the same quantity” will be a steady refrain in our work. In signal processing, “filtering in the frequency domain” is an example of this, where operations are carried out on the spectrum to, in turn, produce a signal in the time domain having desired properties. Another important example of the use of the dual representations is the sampling theorem, which is fundamental in passing between analog and digital signals. In optics, examples are diffraction and interference phenomena, in physics an example is the Heisenberg Uncertainty Principle. In mathematics, celebrated identities in number theory come from Rayleigh’s identity, which, in physics, says that the energy of a signal can be computed in either the time domain or the frequency domain representation.

Underlying much of this development and its wide applicability is the notion of *linearity*. The operations of analyzing a signal into its component parts (taking the Fourier transform) and synthesizing a signal from its component parts (taking the inverse Fourier transform) are *linear operations*, namely integration. The principle of superposition applies to linear systems — the sum of inputs produces a sum of outputs — and one can thus work with a complicated signal by working with linear combinations, sums or integrals, of simpler signals. This is fundamental to all signal processing.

The desire to extend the applicability of Fourier series and the Fourier transform led not only to an increasing array of real-world applications but also to purely mathematical questions. Investigations on many fronts led to a better understanding of approximation and of limiting processes, the nature of integration, linear operators, eigenfunctions, and orthogonality. In extending the range of the mathematical methods it became necessary to move beyond classical ideas about functions and to develop the theory and practice of distributions, also known as generalized functions. The need for such objects was recognized earlier by engineers for very practical applications, and by physicists for computations in quantum mechanics.

In short, in this class there’s a little magic everyday. I want you to see that.

---

**A few comments on audience, backgrounds, and goals** The majority of the students in the class are EEs but not everyone is, and collectively you all bring a great variety of preparation and interests. Students new to the subject are introduced to a set of tools and ideas that are used by engineers and scientists in an astonishing assortment of areas. Students who have seen bits and pieces of Fourier techniques in other courses benefit from a course that puts it all together. I frequently hear this from people who have taken the class. I urge you to keep in mind the different backgrounds and interests and to keep an open mind to the variety of the material, while recognizing that with such a rich panorama we have to be quite selective.

The goals for the course are to gain a facility with *using* the Fourier transform, both specific techniques and general principles, and learning to recognize when, why, and how it is used. Together with a great variety, the subject also has a great coherence, and my hope is that you will appreciate both.

---

**A few comments on math** This is a very mathematical subject no doubt about it – and I am a mathematician – but this is not a mathematics course. What are the distinctions? One is the role of applications. It is true that much of the mathematical development of Fourier analysis has been independent of the growing list of applications in engineering and science, but the mathematicians are missing out on at least some of the fun. Moreover, the applications have proliferated largely because of the computational power now available, and the tilt toward computation is also viewed

suspiciously by maybe a few (not all!) mathematicians. While we will be able to see only a fraction of the problems that call on Fourier analysis, for (almost) every mathematical idea we develop there will be significant applications. This is a feature not always found in courses and textbooks outside of engineering. For example, when speaking of applications in the Preface to his well-known book *Fourier Integrals* (from 1937), the British mathematician E.C. Titchmarsh wrote:

As exercises in the theory I have written out a few of these as it seemed to me that an analyst should. I have retained, as having a certain picturesqueness, some references to “heat”, “radiation”, and so forth; but the interest is purely analytical, and the reader need not know whether such things exist.

Bad attitude.

It is important, however, to at least touch on some of the mathematical issues that arise. Engineering, particularly electrical engineering, draws more and more on mathematical ideas, and does so with increasing sophistication. Those ideas may have come from questions once considered to be solely in the realm of pure mathematics, or may have had their start in real-world problems. This back-and-forth is not new, but the exchange between mathematical ideas and engineering or scientific applications has become more circuitous and sometimes strained. Consider this comment of the Russian mathematician V.I. Arnold in his graduate level book on differential equations, *Geometric Methods in the Theory of Ordinary Differential Equations*:

The axiomatization and algebraization of mathematics, after more than 50 years, has lead to the illegibility of such a large number of mathematical texts that the threat of complete loss of contact with physics and the natural sciences has been realized.

It can be hard for engineers, both students and working professionals, to open an advanced math book and get anywhere. That’s understandable, as per Arnold, but it is limiting. If you look at the current engineering and scientific literature where the Fourier transform is used, it looks pretty mathematical. To take one fairly current example, the subject of *wavelets* (which we won’t do) has become quite important in various parts of signal processing. So too *compressive sensing*. You’ll understand these and other new ideas much better if you know the mathematical infrastructure that we’ll develop. For a list of mathematical topics that we’ll call on see the separate document.

You do need mathematics. You need to understand better the mathematics you’ve already learned, and you need the confidence to learn new things and then to use those new things. If you’re looking for a learning goal, as everyone seems to be doing, that’s a big one. Beyond the specific formulas, facts, etc., I hope that this course offers a dose of mathematical know-how, honest and helpful, and with a light touch.

---

**Topics:** Here are the topics for the course, listed more or less chronologically. Many will be mixed together and will come up in several contexts. This is a big list and we probably won’t get to everything. Let me know your preferences!

1. Periodicity and Fourier series in one-dimension
  - Applications to partial differential equations
  - Orthogonality
2. Fourier transforms
  - Transforms of common signals

- Shifts, scaling, modulation, differentiation
  - Fourier inversion and duality
3. Convolution
    - Applications to filtering, differential equations, probability
  4. Distributions (generalized functions)
    - Delta functions, generalized Fourier transforms
  5. Applications of the delta function
    - Diffraction and the *III* distribution
  6. Sampling and the Nyquist theorem
    - Sampling, interpolation, and aliasing
  7. The discrete Fourier transform (DFT)
    - Properties of the DFT and relationship to the continuous time Fourier transform
    - Discrete convolution and digital filters
    - The FFT algorithm
  8. Linear systems
    - Impulse response, transfer functions for continuous and discrete time systems
    - Linear time-invariant systems
    - Eigenfunctions, eigenvalues
  9. The higher dimensional Fourier transform
    - Higher dimensional complex exponentials and the higher dimensional Fourier transform
    - Lattices and Crystallography
    - Medical imaging, the Radon transform, the projection slice theorem